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Flux Compactifications: Stability and Implications for Cosmology

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Abstract

We study the dynamics of the size of an extra-dimensional manifold stabilised by fluxes. Inspecting the potential for the 4D field associated with this size (the radion), we obtain the conditions under which it can be stabilised and show that stable compactifications on hyperbolic manifolds necessarily have a negative four-dimensional cosmological constant, in contradiction with experimental observations. Assuming compactification on a positively curved (spherical) manifold we find that the radion has a mass of the order of the compactification scale, M_c, and Planck suppressed couplings. We also show that the model becomes unstable and the extra dimensions decompactify when the four-dimensional curvature is higher than a maximum value. This in particular sets an upper bound on the scale of inflation in these models: $V_{\rm max} \sim M_c^2 M_P^2$, independently of whether the radion or other field is responsible for inflation. We comment on other possible contributions to the radion potential as well as finite temperature effects and their impact on the bounds obtained.

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1 Introduction

In recent years great progress has been made in the measurement of the parameters that control the evolution of our universe and experiments like WMAP [1] have provided us with precision data to compare with the predictions of theories of the early universe such as inflation. In this way a standard cosmological model has emerged, the so-called Lambda Cold Dark Matter (ΛCDM) or Concordance Model [2]. This model assumes inflation in the early universe but one of its most surprising features is the small but nonzero observed cosmological constant at present times, that poses a deep fundamental challenge for theorists. There is then a great deal of activity in what could be called cosmological phenomenology, or trying to figure out the implications for cosmology of the available high energy theories of fundamental physics. Many of these hypothesised theories, like String Theory, are formulated in a higher dimensional space, so one commonly assumes that the extra dimensions are compactified with a small volume. The purpose of this letter is to extract cosmological implications of the scenario in which the size of the extra dimensions is stabilised using fluxes in the higher-dimensional space.

Flux compactifications, although an old topic of research in high energy physics (see for instance [3]), have received a lot of attention recently [4]. In the next section we work out in detail the compactification of n extra dimensions in a manifold of constant curvature (hyperbolic or spherical, depending on the sign of the curvature) using fluxes. We will rephrase the dynamics of the size of the extra dimensions as the dynamics of a scalar field (the radion) coupled with gravity in four dimensions. The radion effective potential will enable us to discuss issues of minimisation and stability in section 3 [1]. In particular we will show that only compactifications in spherical spaces allow for a minimum of the radion potential with positive or zero four dimensional cosmological constant, such as the one observed. In case the compact space is hyperbolic, its size can only be stabilised at the cost of a negative 4D cosmological constant. The mass of the radion is of the order of the compactification scale (M_c) in this scenario, as opposed to radion masses of order $m_\phi \sim M_c^2/M_P$, with M_P the four-dimensional reduced Planck mass, used in other studies of radion cosmology [6]. This will change the implications for cosmology of these extra-dimensional scenarios with

[1] See [5] for a related discussion on the stability of these compactifications.

respect to models using other compactification mechanisms that produce a suppressed radion mass [6]. We will also show that, even if the extra dimensions are compactified in a spherical space, there is a maximum possible value for the curvature of the 4D space above which the extra dimensions decompactify, since for higher curvatures the radion effective potential loses its minimum and exhibits a runaway behaviour[2]. This in particular puts a bound on the maximum scale of inflation attainable in these models in terms of the compactification scale: $V_{max} \sim M_c^2 M_P^2$. We comment on other possible contributions to the radion effective potential and how they could modify the bounds obtained within flux compactifications.

2 Compactifications with Fluxes

In this section we review the spontaneous compactification of n extra dimensions using fluxes and a higher dimensional cosmological constant to stabilise the size of the extra dimensions. If, starting in d dimensions, we want to find static solutions that compactify n of them in a manifold of constant curvature, a natural thing to do is to consider a vacuum expectation value for an n- or 4-form field strength (where $d = 4 + n$) [3]. Our starting point is then the following d-dimensional action (we follow Misner, Thorne and Wheeler's book[8] metric and curvature conventions):

$$S = \int d^d x \sqrt{-g} \left(M_*^{n+2} \frac{1}{2} R - \frac{1}{2n!} F_{(\mathbf{n})}^2 - \frac{1}{48} F_{(\mathbf{4})}^2 - \hat{\Lambda} \right), \tag{1}$$

where M_* is the fundamental Planck mass and $\hat{\Lambda}$ is a higher-dimensional cosmological constant. We have just considered pure gravity with a cosmological constant plus a n-form and a 4-form field. This Lagrangian can be seen as a good approximation to study the dynamics of compactification if we assume that all other fields present in our fundamental high energy theory are stabilised with masses higher than those relevant for our study (as we will see, the compactification scale) or do not play any role in the compactification dynamics. The corresponding equations of motion (EOM) resulting from this action read:

$$\partial_M \left(\sqrt{-g} F_{(\mathbf{n})}^{M..Q} \right) = 0, \tag{2}$$

[2]See [7] for a recent study of the decompactification process induced by thermal fluctuations or quantum tunnelling.

$$\partial_M \left(\sqrt{-g} F_{(\mathbf{4})}^{M..Q} \right) = 0, \tag{3}$$

and

$$\begin{aligned} M_*^{n+2} R_{MN} &= T_{MN} - \frac{1}{n+2} g_{MN} T^R{}_R \\ &= -g_{MN} \frac{(n-1)}{n!(n+2)} F_{(\mathbf{n})}^2 + \frac{1}{(n-1)!} F_{(\mathbf{n})M}{}^{P...Q} F_{(\mathbf{n})NP...Q} \\ &- g_{MN} \frac{3}{24(n+2)} F_{(\mathbf{4})}^2 + \frac{1}{6} F_{(\mathbf{4})M}{}^{P...Q} F_{(\mathbf{4})NP...Q} + g_{MN} \frac{2}{n+2} \hat{\Lambda}. \end{aligned} \tag{4}$$

Although we require the existence of static solutions with n compact dimensions, they are not the goal of our study. We will focus on the dynamics of the volume of the extra dimensions. For this we have to obtain the potential for the radion, *i.e.* the lower dimensional field that corresponds to dilatations of the compact dimensions. For obtaining its EOM we will consider the following metric *ansatz*:

$$ds^2 = e^{-\alpha\hat{\phi}(x)} \gamma_{\mu\nu}(x) dx^\mu dx^\nu + e^{-\beta\hat{\phi}(x)} R_0^2 \kappa_{ij} dz^i dz^j, \tag{5}$$

where latin indices run over the n compact dimensions and greek ones run over the uncompactified ones. κ_{ij} is the metric for an Einstein manifold of curvature $s = \pm 1$ (+1 like a sphere or -1 like an hyperbolic plane) and R_0 is a constant with dimension $mass^{-1}$. We fix the constants $\alpha = \sqrt{\frac{2n}{n+2}}$ and $\beta = -\frac{2}{n}\alpha$ in order to get a canonically normalised kinetic term for 4-dimensional gravity and for the radion. The constant R_0 can be fixed to an arbitrary value without loss of generality since, as can be seen from the metric, a change in R_0 is equivalent to a shift in ϕ plus a rescaling of the x^μ coordinates and the form fields. Finally, we consider the following vacuum expectation value (VEV) for the forms:

$$F_{(\mathbf{n})i...j} = \sqrt{\kappa} \hat{B} \; \epsilon_{i...j}, \tag{6}$$

$$F_{(4)\;\mu...\nu} = \sqrt{-\gamma} \hat{E} \mathrm{e}^{-3\alpha\hat{\phi}} \epsilon_{\mu...\nu}, \tag{7}$$

where $\hat{B}$, $\hat{E}$ are constants (of mass dimensions $(4-n)/2$ and $(4+n)/2$, respectively), $\epsilon_{i...j}$ is the totally antisymmetric tensor with n indices and $\epsilon_{\mu...\nu}$ is the totally antisymmetric tensor with 4 indices. The rest of the elements of these forms are zero. Plugging this *ansatz* in the EOM we see that the equations of the forms are satisfied and from the

(i, j) and (μ, ν) components of Einstein equations we get, respectively,

$$M_*^{n+2}\Box_{(\gamma)}\hat{\phi} = \sqrt{\frac{n}{2(n+2)}}\Big[-3\Big(\frac{\hat{B}^2}{R_0^{2n}} + \hat{E}^2\Big)\mathrm{e}^{-3\alpha\hat{\phi}} - 2\Lambda\mathrm{e}^{-\alpha\phi} + \frac{n+2}{n}s\frac{M_*^{n+2}}{R_0^2}\mathrm{e}^{-\frac{n+2}{n}\alpha\hat{\phi}}\Big], \tag{8}$$

and

$$\begin{aligned} M_*^{n+2}R_{\mu\nu}(\gamma) =& M_*^{n+2}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} + \frac{1}{2}\Big[\frac{\hat{B}^2}{R_0^{2n}} + \hat{E}^2\Big]\mathrm{e}^{-3\alpha\hat{\phi}}\gamma_{\mu\nu} \\ &+ \hat{\Lambda}\mathrm{e}^{-\alpha\hat{\phi}}\gamma_{\mu\nu} - \frac{1}{2}s\frac{M_*^{n+2}}{R_0^2}\mathrm{e}^{-\frac{n+2}{n}\alpha\hat{\phi}}\gamma_{\mu\nu}, \end{aligned} \tag{9}$$

where $\Box_{(\gamma)}$ and $R_{\mu\nu}(\gamma)$ are the d'Alembertian and the Ricci tensor computed with the metric $\gamma_{\mu\nu}$. One can check that these EOM can be derived also varying with respect to $\gamma_{\mu\nu}$ and $\hat{\phi}$ the four-dimensional action

$$\begin{aligned} S_{\mathrm{eq}} &= V_n\int \mathrm{d}^4x\,\sqrt{-\gamma}\,\Big[\frac{1}{2}M_*^{n+2}R_{(\gamma)} - \frac{1}{2}M_*^{n+2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} \\ &\qquad - \frac{1}{2}\Big[\frac{\hat{B}^2}{R_0^{2n}} + \hat{E}^2\Big]\mathrm{e}^{-3\alpha\hat{\phi}} - \hat{\Lambda}\mathrm{e}^{-\alpha\hat{\phi}} + \frac{1}{2}s\frac{M_*^{n+2}}{R_0^2}e^{-\frac{n+2}{n}\alpha\hat{\phi}}\Big] \\ &= \int \mathrm{d}^4x\,\sqrt{-\gamma}\,\Big[\frac{1}{2}M_P^2R_{(\gamma)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \\ &\qquad - E^2\mathrm{e}^{-3\alpha\phi/M_P} - \Lambda\mathrm{e}^{-\alpha\phi/M_P} + \frac{1}{2}s\frac{M_P^2}{R_0^2}e^{-\frac{n+2}{n}\alpha\phi/M_P}\Big], \end{aligned} \tag{10}$$

where the volume of the internal dimension is $V_n = \int d^nz\,\sqrt{k}R_0^n$ and in the last two lines we have canonically normalised the action by defining the four-dimensional Planck mass as $M_P^2 = M_*^{n+2}V_n$ and redefining $\hat{\phi} = \phi/M_P$. We have also defined the corresponding "effective four-dimensional flux"

$$E^2 = \frac{V_n}{2}\Big[\frac{\hat{B}^2}{R_0^{2n}} + \hat{E}^2\Big], \tag{11}$$

and a four-dimensional cosmological constant $\Lambda = \hat{\Lambda}V_n$. An important issue in these compactifications is the volume of the extra dimensional manifold, since it enters the definition of all four-dimensional parameters in terms of the higher-dimensional ones. In the hyperbolic case we can construct compact manifolds of constant curvature by modding out the (non-compact) universal covering space of n-dimensional hyperbolic manifolds by certain discrete subgroups of its isometry group [9]. Similarly, in the spherical case, we can simply consider the n-dimensional sphere (since it is already compact) or we can consider other non-trivial topologies by modding out by some

discrete subgroup of its isometry group. In this case the volume of the space constructed in such a way will be $\frac{vol(S^n)}{|\Gamma|}$, where $|\Gamma|$ is the number of elements of the discrete group. For even-dimensional manifolds we only have two possibilities, a sphere and the projective space obtained from the sphere identifying antipodal points, being the volume of the former twice that of the latter. For odd-dimensional spaces we have more possibilities, and in particular we can use the cyclic group Z_q of arbitrary order. For this reason we will leave the volume of the compactification manifold as a free parameter, not related directly with the curvature, but keeping in mind that in the spherical case the maximum possible volume will be the volume of the n-sphere.

Another important point to remark is that the effective 4D action, eq.(10), is *not* the original d-dimensional action with the *ansatze* given by eqs.(5-7) substituted in it. There is a difference in the sign corresponding to the 4-form term $\Delta\mathcal{L} = -\frac{1}{2}V_n\hat{E}^2\mathrm{e}^{-3\alpha\phi}$, since substituting the *ansatze* eq.(5-7) naively in the action would have resulted in a contribution to this effective action given by $\Delta\mathcal{L} = \frac{1}{2}V_n\hat{E}^2\mathrm{e}^{-3\alpha\phi}$, that gives the wrong EOM (see [10]). The reason for this is the following: if we substitute in an action the VEV of the derivative of a field in terms of other fields (as we have made for the forms, since $F_{\mathbf{4}} = dA_{\mathbf{3}} = \frac{E}{\sqrt{-\gamma}}\mathrm{e}^{-3\alpha\phi(x)}\epsilon_{\mu\ldots\nu}$) and we vary the action with respect to these fields ($\phi(x)$ in our case) we are not going to obtain the correct EOM, since in the original action we were varying with respect to $A_{\mathbf{3}}$, and given that this field appears with a derivative in that term, and ϕ does not, such a procedure is not justified[3]. The action (10), although does *not* come from the higher dimensional one substituting the particular *ansatze* we have chosen, does produce the correct EOM, completely equivalent to the higher-dimensional ones.

3 Radion Dynamics and Inflation

As we have shown in the previous section, the dynamics of the volume modulus in flux compactifications is reduced to the dynamics of a scalar field in curved space with the following potential

$$V(\phi) = E^2 e^{-3\alpha\phi/M_P} + \Lambda e^{-\alpha\phi/M_P} - \frac{1}{2}s\frac{M_P^2}{R_0^2}e^{-\frac{n+2}{n}\alpha\phi/M_P}. \tag{12}$$

Several interesting points can be highlighted by inspecting this potential. First, as was discussed in ref.[7], $V \to 0$ for $\phi \to \infty$, so in case the potential has a minimum with

[3]We thank J.A. Casas for clarifying this point to us.

$V > 0$ it is necessarily metastable[4], but in any case the lifetime of the decay through quantum tunneling can be easily made bigger that the age of the universe. However, it is not guaranteed that there is a minimum at finite ϕ. For this to happen the following conditions have to be satisfied

$$\Lambda \leq \Lambda_{\max} \equiv 3(n-1)E^2 \left(\frac{n+2}{6n^2} \frac{M_P^2}{E^2 R_0^2} \right)^{\frac{n}{n-1}}, \tag{13}$$

for $s = +1$ and

$$\Lambda \leq 0, \tag{14}$$

for $s = -1$. Assuming the corresponding condition is fulfilled, *i.e.* there is a finite value ϕ_0 such that $V'(\phi_0) = 0$, we have, for the effective four-dimensional cosmological constant

$$V(\phi_0) = -2E^2 e^{-3\alpha\phi_0/M_P} + \frac{s}{n} \frac{M_P^2}{R_0^2} e^{-\frac{n+2}{n}\alpha\phi_0/M_P}, \tag{15}$$

which immediately shows that compactification on a hyperbolic manifold ($s = -1$) leads to a negative cosmological constant in the lower dimensional theory. Compactifications on positively curved manifolds can on the other hand result on a four-dimensional cosmological constant of any sign, depending on the value of the higher-dimensional one. If it is tuned against the fluxes as

$$\Lambda = \Lambda_0 \equiv (n-1)E^2 \left(\frac{M_P^2}{2nE^2 R_0^2} \right)^{\frac{n}{n-1}}, \tag{16}$$

the four-dimensional cosmological constant vanishes, being positive or negative when Λ is larger or smaller than Λ_0, respectively. The above conclusions strongly rely on the fact that the flux contribution to the radion potential has a well defined sign. It is possible to obtain new contributions to the radion potential with the opposite sign, for instance those coming from a wrapped $p-$brane (with $3 \leq p \leq 2+n$ since the contribution of a $n+3$ brane is equivalent to that of $\hat{\Lambda}$) that would give a contribution to the radion potential proportional to its tension (see for instance [7]),

$$V_p(\phi) = T_p e^{-\frac{2n+3-p}{n}\alpha\phi/M_P}, \tag{17}$$

where we have conveniently absorbed in the definition of the brane tension, T_p, the the volume and other numerical factors appearing in the dimensional reduction. It

[4] In [7] it was argued that this property applies to a wider class of compactifications, using not only fluxes in order to generate the radion potential, but also branes and non-perturbative effects.

is of course not guaranteed that the introduction of those branes will not modify the background in an important way. In fact, one would expect that the internal manifold would no longer be Einstein but acquire warping. Furthermore, singularities will in general appear in the classical description of the background [11]. However, for the sake of the present argument we will assume that the backreaction of the branes can be neglected so that their only effect is to add a contribution to the radion potential like the one in eq.(17). The exponent shows that this term is irrelevant for $\phi \to \pm\infty$, provided fluxes and a higher-dimensional cosmological constant are present. Its generic effect is therefore to increase or decrease the potential at intermediate values of ϕ for positive and negative tensions, respectively. This means that positive tension branes can be used to increase the effective four-dimensional cosmological constant (see [12] for a recent application to uplift AdS to dS vacuum in string theory). Nonetheless one should be cautious when doing this since too large a positive contribution can ruin the existence of a minimum for the radion potential and therefore destabilise the extra dimensions (see also [13]). Negative tension branes on the other hand, tend to decrease the effective four-dimensional cosmological constant but they also tend to stabilise the radion potential. In particular it is possible to find values of the parameters such that the radion is stabilised with hyperbolic compact dimensions and a positive four-dimensional cosmological constant but only at the cost of introducing these branes with *negative* tension, (the potential would be unstable without the branes, that decrease it in the intermediate region so that it develops a minimum before the value of the potential crosses zero), as can be seen from the value of the potential at the minimum including the effect of the branes

$$V(\phi_0) = -2E^2 e^{-3\alpha\phi_0/M_P} - \frac{1}{n}\frac{M_P^2}{R_0^2} e^{-\frac{n+2}{n}\alpha\phi_0/M_P} - \frac{n+3-p}{n} T_p e^{-\frac{2n+3-p}{n}\alpha\phi_0/M_P}. \qquad (18)$$

Given the fact that negative tension branes usually suffer from severe stability problems and the possible important effects of the backreaction of the branes on the background, we prefer not to pursue this possibility any further, but restrict ourselves to the potential in eq.(12) arising exclusively from fluxes and a higher-dimensional cosmological constant. (A review of the recent attemps to obtain moduli stabilization in string theory can be found in [14].) In that case, the phenomenological restriction of having a positive (and eventually small) four-dimensional cosmological constant prevents the use of hyperbolic compactifications that we will not consider any more. All the qualita-

tively different behaviours for the radion that can be obtained in flux compactifications on a positively curved manifold are graphically summarised in fig. 1, where we have displayed the radion potential for fixed values of E and R_0 and the following values of Λ, $\Lambda = 0.9\Lambda_0, \Lambda_0, 1.1\Lambda_0, \Lambda_{\rm max}$ and $1.1\Lambda_{\rm max}$, corresponding to stable extra dimensions with negative, zero and positive effective four-dimensional cosmological constant, respectively the first three, and marginally stable and unstable the last two. In par-

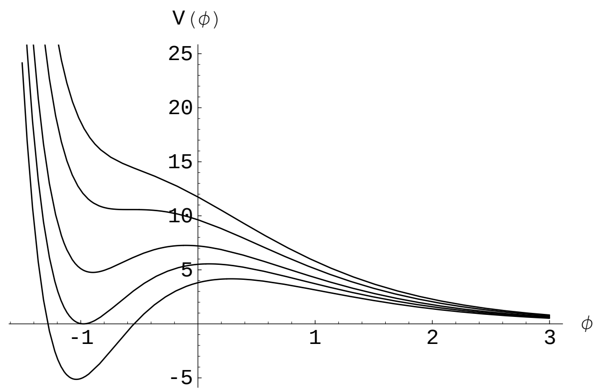

Figure 1: Radion potential for different values of Λ with fixed E and R_0. The different lines are for Λ equal to, from top to bottom, $1.1\Lambda_{\rm max}$, $\Lambda_{\rm max}$, $1.1\Lambda_0$, Λ_0 and $0.9\Lambda_0$, corresponding to unstable and marginally stable extra dimensions the first two and stable extra dimensions with positive, zero and negative effective four-dimensional cosmological constant the lower three.

ticular, using the fact that $0 \leq \Lambda_0 \leq \Lambda_{\rm max}$, we see how, starting with the radion on a stable or metastable minimum, any contribution that increases the higher-dimensional cosmological constant can destabilise the system leading to a run-away potential for the radion as in the top plot in the figure. In that case the extra dimensions would decompactify, rendering the model unrealistic. This possibility will allow us to put stringent bounds on the scale of inflation, using the fact that the couplings of the radion are completely determined by general covariance[5].

In order to obtain a realistic situation we consider that the higher-dimensional cosmological constant is tuned to give a vanishing four-dimensional cosmological constant,

[5]It can be argued that radiative corrections arising below the compactification scale could spoil the form of the radion potential that ultimately comes from 6D general covariance. However, even if these corrections are present one would expect them to be of order $\delta V \sim M_c^4 f_1(\phi/M_P)$, that represent subdominant corrections with respect to the terms we are already considering in the potential that are of order $V \sim M_c^2 M_P^2 f_2(\phi/M_P)$, where f_1 and f_2 are dimensionless functions.

$\Lambda = \Lambda_0$. The compactification scale can then be written as

$$M_c^2 = \frac{1}{R_0^2} e^{-\frac{n+2}{n}\alpha\phi_0} = \frac{1}{R_0^2}\left(\frac{M_P^2}{2nE^2R_0^2}\right)^{\frac{n+2}{2(n-1)}}, \tag{19}$$

the different Kaluza-Klein modes appearing in our theory will have masses proportional to this compactification scale. The precise value depends on the topology of our compact space, so they could be significantly higher. We can use this expression of the compactification scale to trade R_0 for it in all our formulae, writing everything as a function of E, M_P and M_c. For instance, our tuned cosmological constant reads

$$\Lambda_0 = (n-1)\left(\frac{M_P^2 M_c^2 E}{2n}\right)^{2/3}, \tag{20}$$

and similarly for the critical value of the cosmological constant for a stabilised radion that can be written

$$\Lambda_{\max} = 3(n-1)\left(\frac{n+2}{3n}\right)^{\frac{n}{n-1}}\left(\frac{M_P^2 M_c^2 E}{2n}\right)^{2/3}. \tag{21}$$

The radion mass, given by the second derivative of its potential at the minimum, turns out to be of the order of the compactification scale,

$$m_\phi^2 = V''(\phi_0) = 4\frac{n-1}{n(n+2)}M_c^2. \tag{22}$$

This mass is much larger than the normally used estimates that suppress it by the ratio M_c/M_P. This last estimate is usually correct when the extra dimensions are flat and the radion is massless at leading order (see for instance [15]). In our case, however, the fluxes and higher-dimensional cosmological constant compactify the extra dimensions in a highly curved space and therefore give a mass to the radion that is much larger than the naive estimate[6]. This is very interesting because a Planck suppressed mass for the radion typically produces very stringent cosmological constraints [6], constraints that are easily eluded in our case. Using this value of the radion mass and the fact that its couplings are Planck suppressed, we can estimate its decay width as

$$\Gamma_\phi = \tau_\phi^{-1} \sim \frac{m_\phi^3}{M_P^2} \sim \frac{M_c^3}{M_P^2}. \tag{23}$$

[6]This does not mean that in these kind of compactifications there could not be some light fields with mass $m \sim M_c^2/M_P$. For instance in the SUGRA of ref.[16] two extra dimensions can be compactified in a sphere using a magnetic field but there is a linear combination of the radion and dilaton that remains massless after compactification. It has been argued that a suppressed mass for this field is generated after SUSY is broken [17]. We are assuming that no such light field is present here.

This indicates that the radion decays before BBN for $M_c \gtrsim 10$ TeV, decays after BBN for 10 MeV $\lesssim M_c \lesssim 10$ TeV and is effectively stable for $M_c \lesssim 10$ MeV. In the last two cases, constraints on the compactification scale could arise from modifications of the successful predictions of BBN or the CMB spectrum (see [18]) and over-closure of the universe, respectively. A more detailed study of such possibilities is deferred to future work.

The last quantity relevant for our discussion is the value of the potential at the critical point, where it has a saddle point instead of a minimum when $\Lambda = \Lambda_{\max}$, (second from the top in fig. 1),

$$V_{max} = V(\phi_{\max}, \Lambda_{\max}) = 2\frac{n-1}{n(n+2)}\left(\frac{n+2}{3n}\right)^{\frac{3n}{2(n-1)}} M_P^2 M_c^2, \tag{24}$$

where we have denoted by $\phi_{\max}$ the value of the radion such that $V'(\phi_{\max}) = V''(\phi_{\max}) = 0$ for $\Lambda = \Lambda_{\max}$. Although the scale of the 4D effective potential at this critical point ($V_{max}^{1/4}$) is above the compactification scale (and can be even above the higher dimensional Planck mass M_*), the use of a field theoretic 4D description is fully justified. The destabilisation occurs when the curvature of the compact dimensions and the four non-compact ones are roughly of the same order, $\sim M_c^2$, that we assume to be well below the higher dimensional fundamental mass (so that higher order curvature corrections are negligible). And for the 4D description to be a good approximation we just have to make sure that the temperature is always below the compactification scale, so KK excitations will not be produced.

We are now in a position to discuss the phenomenological implications of flux compactifications on positively curved internal manifolds for inflation. We consider that the higher-dimensional cosmological constant has been tuned to give a vanishing effective four-dimensional constant at the minimum of the potential, $\Lambda = \Lambda_0$. The first possibility is to consider the radion as the inflaton itself. Of course a detailed study is necessary in order to determine the viability of the model although in principle it seems plausible that enough number of e-foldings can be obtained by appropriately tuning the initial position of the radion as close as necessary to its maximum. In this case it is clear that we can not obtain inflation scales higher than the maximum of the $\Lambda = \Lambda_0$ curve in fig.1 (the one with the minimum in $V = 0$).

The other obvious possibility is considering that inflation is driven by *any* other scalar field but the radion. In this case we can get higher inflation scales, but when

getting bounds on this scale the interesting thing is that it does not really matter the details of inflation. What matters is the fact that during inflation the slow-roll condition ensures that the inflaton potential is almost constant so the only effect on the radion potential is just to add a contribution to the higher-dimensional cosmological constant equal to the inflaton potential:

$$\Lambda \to \Lambda + V(\chi), \tag{25}$$

where $V(\chi)$ is the inflaton potential, that can be considered constant as long as the slow-roll condition (along the χ direction) holds. It is then evident from our previous discussion that for

$$V(\chi) \geq \Lambda_{\max} - \Lambda_0, \tag{26}$$

the radion potential loses its minimum and the extra dimensions decompactify. This inequality is precise only when the slow-roll condition for the inflaton potential is maintained but can be considered as a reasonable order of magnitude estimate for realistic situations. Of course, the details of the inflaton potential and the initial conditions are required for a detailed computation of the scale at which our theory decompactifies. However, the fact that the very slow-roll condition can be jeopardised if the inflaton potential is higher that this maximum scale (since the slope would be much larger than expected in the radion direction of the two-dimensional potential), indicates that eq.(24) is a conservative estimate for the maximum scale of inflation, where we have assumed that the extra dimensions stabilise before inflation takes place. This bound can have important consequences for many models of inflation in extra-dimensional theories. For instance, in the recent study of inflation in 6D gauged supergravity [19] the authors consider the possibility of chaotic inflation in an anomaly free $N = 1$ gauged supergravity in six dimensions [20], compactified with fluxes on a two-sphere. Their chaotic potential is, during inflation, of order

$$V_{ch} \sim M_c^2 M_P^2, \tag{27}$$

which is on the verge of the decompactification limit. This is a clear example in which the possibility of destabilization of the extra dimensions during inflation has to be carefully taken into account in order to determine the phenomenological viability of the model.

The bound we have found on the maximum scale of inflation seems to disfavour low scale compactifications, since the CMB data prefers high inflation scales:

$$\left(\frac{V_{infl}}{\epsilon}\right)^{1/4} \sim 10^{16} GeV, \tag{28}$$

where $\epsilon = M_P^2(V'/V)^2$ is the slow roll parameter and this is valid only when the inflaton is responsible for the density perturbations (see [21] and references therein). As an example of the bounds implied, for $n = 2$ extra dimensions of size $M_c \sim 10^{-3}$ eV, the maximum allowed scale of inflation is $\sim$ TeV^4, whereas for TeV-sized extra dimensions we obtain $V_{\max} \sim (10^{10} - 10^{11}\ \text{GeV})^4$.

Another effect that could be important when describing the stability of flux compactifications is finite temperature corrections. It has been very recently shown in [22] that, due to the fact that gauge couplings are dilaton dependent, finite temperature effects $\delta V \propto g^2 T^4$, can destabilise the dilaton potential for temperatures above $10^{11} - 10^{12}$ GeV. A similar effect can also affect the radion potential since, as can be easily seen from the kinetic term of a bulk gauge boson, in our case the gauge couplings are radion dependent

$$g^2 = \frac{g_d^2}{V_n} e^{-\alpha\,\phi/M_P}, \tag{29}$$

where g_d is the higher-dimensional gauge coupling. Therefore we see that the finite temperature contribution to the radion potential scales as the cosmological constant term and thus this effect can be taken into account by simply replacing

$$\Lambda \to \Lambda + \xi T^4, \tag{30}$$

where the parameter ξ is a number of order one that depends on the number of interacting species. This leads to a maximum temperature in the early universe of the order of

$$T^4_{\max} \sim \Lambda_{\max} - \Lambda_0 \sim (M_c^2 M_P^2 E)^{2/3}. \tag{31}$$

Note that, unless $E \leq M_c^4/M_P^2$, this temperature is much larger than the compactification scale and therefore, beyond the validity of our four-dimensional approximation. A full higher-dimensional study should then be performed to give precise bounds on the maximum attainable temperature from decompactification. Nonetheless, naively one would expect that the higher number of active species when the Kaluza-Klein modes are excited would increase the finite temperature correction to the radion effective potential, so the bound given by eq.(31) could be considered a conservative one.

As a final remark we would like to mention that a similar bound as the one we have placed on the maximum scale of inflation in flux compactifications applies to a much more general range of compactifications. It has been recently argued in [7] that not only fluxes or a higher dimensional cosmological constant but also space-filling branes, non-perturbative effects or higher order string corrections lead in string theory to a qualitatively similar potential to the one we have depicted in fig. 1. The effect of branes has already been discussed in the previous section. Non-perturbative effects are argued in [12] to give rise to the following potential

$$\delta V_{NP} \sim B_{NP} e^{-2ae^{\frac{2}{3}\alpha\phi/M_P}} e^{-\frac{2}{3}\alpha\phi/M_P}, \tag{32}$$

where a is here a parameter that depends on the mechanism to generate such corrections, whereas higher order string corrections give a contribution [23]

$$\delta V_{HO} \sim B_{\alpha'} e^{-3\alpha\phi/M_P}. \tag{33}$$

Non-perturbative effects are again irrelevant for $\phi \to \pm\infty$ so their effect is similar to the one of $p-$branes in the sense that they only modify the intermediate regions of the potential, while the effect of higher order string corrections is similar to that of the fluxes and can be accounted for by simply shifting the constant E^2 in our analysis. In any case, the relevant point is that these corrections do not modify in a qualitative way the radion potential. Thus, a positive contribution to the cosmological constant (like a slow-rolling inflaton) will tend to remove the (meta)stable minimum for the radion potential and therefore a bound on the maximum curvature during inflation similar to the one we have discussed can be put, although the details will of course depend on the sources for radion stabilisation used.

4 Conclusions

In this letter we have studied the dynamics of the size of the extra dimensions (the radion) in flux compactifications. We have obtained the radion effective potential and extracted some interesting results from it. First, we have shown that the requirement that there is a minimum with positive or zero 4D cosmological constant implies that the compactification manifold has spherical curvature, hyperbolic compactifications being ruled out if fluxes are responsible for the stabilisation of the extra-dimensional

volume. Then, we have seen that there is a maximum value for the de Sitter curvature of the 4D space for which the radion can be stabilised, and this sets an upper bound on the scale of inflation from decompactification of the extra dimensions. This maximum value occurs when the 4D space and the extra dimensions have roughly the same curvature $\sim M_c^2$, so the limit on the scale of inflation is of order $V_{max} \sim M_c^2 M_p^2$. For the extreme ADD case with $M_c \sim 10^{-3}$ eV and $n = 2$ the maximum scale of inflation is $\mathcal{O}$(TeV), while for TeV size extra dimensions this bound implies that the maximum scale is $\mathcal{O}(10^{10} - 10^{11}\text{GeV})$. This bound seems to disfavour models with a low compactification scale, since CMB data prefers high inflation scales (at least when the inflaton is responsible for the density perturbations). Finally, we have briefly mentioned other possible contributions to the radion potential. Finite temperature effects, that can act as an effective cosmological constant and therefore destabilise the radion have been shown to be negligible within the range of validity of our effective four-dimensional description. Other sources that may appear in string theory, like wrapped $p-$branes, non-perturbative effects of higher order string corrections do not qualitatively change the picture, still existing a maximum value of the inflation scale above which the extra dimensions would decompactify, although the details of such scale depend on the particular contributions.

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